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Provisional Application for Patent Cover Sheet

To: MS Provisional Patent Application
Commissioner for Patents
P.O. Box 1450
Alexndria, VA 22313-1450

Sir: Enclosed for filing please find the following provisional patent application:
Title: A Method For Transforming a Digital Signal From the Time Domain to the Frequency Domain

Enclosed please also find the following papers: Serial Number: ____________

☒ 18 pages Specification; 5 pages claims; 0 pages abstract Filing Date September 29, 2003

☒ 3 sheets of drawings

☐ Power of Attorney

☐ Assignment of the invention

☐ A Verified Statement Claiming Small Entity Status

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☐ Applicant claims small entity status.

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A Method for Transforming a Digital Signal from the Time Domain to the Frequency Domain

Background of the Invention

Reversible transforms that map integers to integers are important tools for state-of-the-art lossless coding schemes. For integer inputs, such a transform generates integer outputs that approximate its floating-point prototype transform outputs. The original integer inputs can be completely recovered by passing the integer outputs from the forward transform through the inverse transform.

A transform called integer modified discrete cosine transform (IntMDCT) is recently proposed and used in the ISO/IEC MPEG-4 audio compression. The IntMDCT can be derived from its prototype – the modified discrete cosines transform (MDCT). In the book [1], Malvar gave an efficient realization of MDCT by cascading a bank of Givens rotations with a DCT-IV block. It is well known that Givens rotation can be factorized into three lifting steps for mapping integers to integers [2]. Therefore, the realization of IntMDCT relies on an efficient implementation of integer DCT-IV.

Integer transforms can be directly converted from their prototypes by replacing each Givens rotation with three lifting steps. Because in each lifting step there is one rounding operation, the total rounding number of an integer transform is three times the Givens rotation number of the prototype transform. For discrete trigonometric transforms (Discrete Fourier Transform (DFT), Discrete Cosine Transform (DCT), Discrete Hartley Transform (DHT), and Discrete W Transform (DWT)), the number of Givens rotations involved is usually at $N\log_2 N$ level, where N is the block size. Accordingly, the total rounding number is also at $N\log_2 N$ level for the family of directly converted integer transforms. Because of the roundings, an integer transform only approximates its floating-point prototype. The approximation error increases with the number of roundings.

In this invention, a method for transforming a digital signal from the time domain to the frequency domain, in particular reversible integer transform like DCT IV, DWT IV and other transforms. The total rounding number of the method according to the invention can be significantly reduced, for example in case of DCT IV to be as low as *2.5N*. As a result, the approximation error of the method according to the invention is far less than that of the directly converted integer transforms. The computational complexity of the proposed method is also very low.

Summary of the Invention

The invention relates to a method for transforming a digital signal from the time domain to the frequency domain. The method can be used for any types of digital signal, such as audio, image or video signals. The digital signal, which corresponds to a physical measured signal, may be generated by scanning at least a characteristic feature of a corresponding analog signal (for example, the luminance and chrominance values of a video signal, the amplitude of an analog sound signal, or the analog sensing signal from a sensor). The digital signal comprises a plurality of data symbols. The data symbols of the digital signal are grouped into blocks, with each block having the same predefined number of data symbols based on the sampling rate of the corresponding analog signal.

The digital signal is transformed to the frequency domain by a transforming element based on a transformation function comprising a transformation matrix. In this method according to the invention, each block of the digital signal is divided into two sub-blocks with equal number of data symbols. The transforming element transforms the digital signal block by block by processing, for each block, the two sub-blocks of data symbols as input signals simultaneously to generate two corresponding sub-blocks of transformed output signal, which can be combined to form a corresponding block of an output signal. It should be noted that the block size of the input signals and the output signals are the same.

The transforming element includes a plurality of lifting stages. A first lifting stage receives the two sub-blocks as first and second input signals, respectively. The first input

signal is processed in a transformation path in the first lifting stage which includes a domain transformer, a rounding unit and a summation unit. The domain transformer can be any type of non-integer (real value) transformation function. The transformed first input signal is summed with the second input signal and result in an output signal. The output signal and the first input signal of the first lifting stage are used as the first and second input signals for a second (or subsequent) lifting stage, resulting in an output signal by the second lifting stage. Similarly, the output signal and the first input signal of the second lifting stage are used as the first and second input signals for another subsequent lifting stage. Since the output signal and the first input signal from each stage are received by the subsequent stage, these two signals shall be considered to be signals output from each of the lifting stages, although the second input signal of the subsequent stage is identical to the first input signal of the previous stage (i.e. not transformed).

The transforming element can be illustrated based on the model of a lifting ladder. The lifting ladder model has two side pieces, each for receiving one of the two sub-blocks of data symbols. Two or more cascading lifting stages are provided between the two side pieces. Each lifting stage receives a signal at one end (input end), and outputs a signal at the other end (output end) via a summation unit. A domain transformer is arranged at the input end of the lifting stage, and a rounding unit is arranged at the output end, between the domain transformer and the summation unit. The lifting stages are arranged between the side pieces in an alternating manner, such that the output (or input) ends of adjacent lifting stages are connected to the different side pieces.

The number of lifting stages of the transforming element is defined by the number of lifting matrices which is determined by a process according to this invention as described in the following.

This process first factorizes the transformation matrix of the transformation function into a string of first lifting matrices. Each lifting matrix comprises four sub-matrices, with two permutation matrices as two of the sub-matrices in one diagonal, and a zero and any general matrix as the other two of the sub-matrices in the other diagonal. A permutation

matrix is a matrix which changes the position of the elements in another matrix. An example of a lifting matrix is:

$$\begin{bmatrix} P2_N & 0 \\ G_N & P1_N \end{bmatrix}$$

wherein G_N is a general matrix, and $P1_N$ and $P2_N$ are permutation matrices. The permutation matrices may also be identical. Also, Identity Matrix is commonly used as a form of permutation matrix. An Identity Matrix is a matrix shown in the following:

$$I_N = \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & 1 \end{bmatrix}$$

The general matrix G_N may take the form of any matrix configuration, for example any transformation matrix, with order N.

Subsequently, the process identifies and combines any adjacent pairs of the first lifting matrices in said string having their permutation matrices and the general matrix at the same sub-matrices position in each lifting matrix. Considering the following example:

$$\begin{bmatrix} P2_N & 0 \\ A & P1_N \end{bmatrix} \cdot \begin{bmatrix} P4_N & 0 \\ B & P3_N \end{bmatrix} = \begin{bmatrix} P2_N \cdot P4_N & 0 \\ A \cdot P4_N + P1_N \cdot B & P1_N \cdot P3_N \end{bmatrix}$$, wherein A and B are any general matrix. The two first lifting matrices (left side of the above equation) can be combined to form one second lifting matrix (right side of the equation), as the permutation matrices and the general matrix of both the first lifting matrices are in the same position. It should be noted that the product of two permutation matrices will also result in a permutation matrix.

As a result, a string of second lifting matrices is generated. It should be noted that the string of second lifting matrices may comprise a mixture of lifting matrices which are identical to the first lifting matrices and matrices which are a result of combination of first lifting matrices. The number of second lifting matrices then corresponds to the number of lifting stages in the lifting ladder of the transforming element.

It should be noted that although the transformation element is described in the form of the lifting ladder model, it is only to illustrate the transformation paths of the transformation element. However, the invention shall not be limited to said ladder model.

Figure 1 shows a flow chart of an embodiment of the method according to the invention wherein the transformation matrix of the transformation function is converted by the above process into five second lifting matrices, resulting in the use of five lifting stages. From Fig.1, x_1 and x_2 are first and second sub-blocks of a block of the digital signal, respectively. z_1, z_2 and z_3 are intermediate signals, and y_1 and y_2 are output signals corresponding to the respective first and second sub-blocks.

The method according to the invention can be used for transforming an input digital signal which represents integer values to an output signal which also represents integer values. In other words, a "same word-length" transformation is performed such that word-lengths of the input signal and the output signal are the same (for example, a 8-bit input data is transformed to a 8-bit output data). Since both the input and output signals have the same word-length, the transformation method according to the invention is reversible. The output signal may be transformed back to the original input signal by performing the transformation method according to the invention. Such a reversibility property of the transformation according to the method of the invention can be used in lossless coding in which the output signal should be identical to the original input signal.

Due to this reversibility property, the invention shall also include the transforming of a digital signal from the frequency domain to the time domain using the transformation element as described earlier.

Such a same word-length (or integer) transformation of signals according to the invention can be used in many applications and systems such as MPEG audio, image and video compression, JPEG2000 or spectral analyzers (for analyzing Infrared, Ultra-violet or Nuclear Magnetic Radiation signals). It can also be easily implemented in hardware systems such as in a fixed-point Digital Signal Processor (DSP), without having to consider factors such as overflow in the case of a real-value signal transformation.

As can be seen from the method according to the invention, all the data symbols in each block of the digital signal are provided to the transforming element as a data vector. In each lifting stage, the data vector is transformed in the domain transformer, and the transformed data vector is rounded subsequently to an integer vector (i.e. after the transformation in each lifting stage). In other words, rounding is performed in the method of the invention once on the data vector as a whole. This is in contrast to any method according to the state of the art, wherein the rounding process is performed within the transformation process for the individual element or data symbol in each block of the digital signal. Thus, the number of rounding operations in the method according to the invention is greatly reduced. Due to the reduced number of rounding operations, the method according to the invention does not require large computation time and computer resource.

Discrete Cosine Transforms, Discrete Sine Transforms Discrete Fourier Transforms or Discrete Wavelet Transforms are examples of transformation functions that may be used as the transformation function according to the invention. It should be noted that the number of lifting stages of the transforming element may be different, depending on the result of the above described process carried out on the respective transformation functions.

The factorizing of the transformation matrix into the string of the first lifting matrices may include decomposing the transformation matrix into a string of transformation sub-matrices which then in turn are decomposed into said string of first lifting matrices.

The corresponding transformation matrix of the transformation function is in general decomposed into three transformation sub-matrices. Such is the case, for example when DCT-IV or DWT-IV is the transformation function, wherein the detail implementation of both will be described in detail later. Each of the transformation sub-matrices are then further factorized or decomposed into three lifting matrices, thereby forming nine lifting matrices as the string of first lifting matrices. As already described above, the nine lifting matrices are further combined, if possible, to form the string of second matrices. However, in some cases, less than nine lifting matrices forms the string of first lifting matrices when one or more of the transformation sub-matrices is a permutation matrix.

For the case with DCT-IV as the transformation function, the three transformation sub-matrices corresponds to a pre-rotation matrix, a DCT-IV transformation-based matrix and a post-rotation matrix. In the special example, the DCT-IV transformation-based matrix and the post-rotation matrix can be combined to form a T matrix. Thus the factorization of the pre-rotation matrix and the T matrix only generate six lifting matrices as the string of first lifting matrices. As can be seen in one of the examples later, the last lifting matrix of the pre-rotation matrix is combined with the first lifting matrix of the T matrix, resulting in a total of five lifting matrices in the string of second lifting matrices, the first two and the last two second lifting matrices are identical to those of the first lifting matrices.

For the case with DWT-IV as the transformation function, the three transformation sub-matrices corresponds to a rotation matrix, a DWT-IV transformation-based matrix and a permutation matrix. In this case, the permutation matrix is not factorized into lifting matrices, and is implemented as a permutation block which rearranges the order of the elements of the input signal. Thus only the rotation matrix and the DWT-IV transformation-based matrix are factorized, generating six lifting matrices as the string of first lifting matrices. As can be seen again in one of the examples later, the last lifting matrix of the rotation matrix is combined with the first lifting matrix of the DWT-IV transformation-based matrix, resulting in a total of five lifting matrices in the string of

second lifting matrices, the first two and the last two second lifting matrices are identical to those of the first lifting matrices.

It should be noted that each of the elements in the lifting matrices is a matrix itself.

The invention not only relates to a method for transforming a digital signal from the time domain to the frequency domain (and vice versa), but also includes a computer program, a computer readable medium and a device for implementing the said method.

Example of the method according to the invention based on an Integer-type DCT IV as the transformation function.

An example of the method according to the invention is illustrated using the DCT-IV transformation function. The DCT-IV of a N-point real input sequence $x(n)$ is defined as follows [2]:

$$y(m) = \sqrt{\frac{2}{N}} \sum_{n=0}^{N-1} x(n) \cos\left(\frac{(m+1/2)(n+1/2)\pi}{N}\right) \quad m,n = 0,1,\cdots,N-1 \tag{1}$$

Let C_N^{IV} be the transformation matrix of DCT-IV, that is

$$C_N^{IV} = \sqrt{\frac{2}{N}} \left[\cos\left(\frac{(m+1/2)(n+1/2)\pi}{N}\right)\right]_{m,n=0,1,\cdots,N-1} \tag{2}$$

The DCT-IV matrix can be factorized into the following form:

$$C_N^{IV} = R_{po} \begin{bmatrix} C_{N/2}^{IV} & \\ & C_{N/2}^{IV} D_{N/2} \end{bmatrix} R_{pr} \tag{3}$$

where R_{pr} is a $N \times N$ pre-rotation matrix defined by

$$R_{pr} = \frac{1}{\sqrt{2}} \begin{bmatrix} I_{N/2} & I_{N/2} \\ I_{N/2} & -I_{N/2} \end{bmatrix} \tag{4}$$

and $I_{N/2}$ is the identity matrix of order *N/2*.

$C_{N/2}^{IV}$ is the DCT-IV matrix of order *N/2* and $D_{N/2}$ is an order *N/2* diagonal matrix given by

$$D = \begin{bmatrix} 1 & & & & \\ & -1 & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & -1 \end{bmatrix} \tag{5}$$

R_{po} is a $N \times N$ post-rotation matrix defined by

$$R_{po} = \begin{bmatrix} \cos\frac{\pi}{4N} & & & & & & \sin\frac{\pi}{4N} \\ & \cos\frac{3\pi}{4N} & & & & \sin\frac{3\pi}{4N} & \\ & & \ddots & & ⋰ & & \\ & & & \cos\frac{(N-1)\pi}{4N} \quad \sin\frac{(N-1)\pi}{4N} & & & \\ & & & -\sin\frac{(N-1)\pi}{4N} \quad \cos\frac{(N-1)\pi}{4N} & & & \\ & & ⋰ & & \ddots & & \\ & -\sin\frac{3\pi}{4N} & & & & \cos\frac{3\pi}{4N} & \\ -\sin\frac{\pi}{4N} & & & & & & \cos\frac{\pi}{4N} \end{bmatrix} \tag{6}$$

Let

$$T = \begin{bmatrix} C_{N/2}^{IV} & \\ & C_{N/2}^{IV} D_{N/2} \end{bmatrix} R_{pr} = \frac{1}{\sqrt{2}} \begin{bmatrix} C_{N/2}^{IV} & C_{N/2}^{IV} \\ C_{N/2}^{IV} D_{N/2} & -C_{N/2}^{IV} D_{N/2} \end{bmatrix} \tag{7}$$

Equation (3) is then expressed as

$$C_N^{IV} = R_{po} T \tag{8}$$

Both R_{po} and T can be factorized into the product of three lifting matrices:

$$T = T_1 T_2 T_3 = \begin{bmatrix} I_{N/2} & \\ K_1 & I_{N/2} \end{bmatrix} \cdot \begin{bmatrix} -D_{N/2} & K_2 \\ & I_{N/2} \end{bmatrix} \cdot \begin{bmatrix} I_{N/2} & \\ K_3 & I_{N/2} \end{bmatrix} \tag{9}$$

where $K_1 = -\left(C_{N/2}^{IV} D_{N/2} + \sqrt{2} I_{N/2}\right) C_{N/2}^{IV}$ $K_2 = \frac{C_{N/2}^{IV}}{\sqrt{2}}$ $K_3 = \sqrt{2} C_{N/2}^{IV} D_{N/2} + I_{N/2}$, and

$$R_{po} = R_1 R_2 R_3 = \begin{bmatrix} I_{N/2} & \\ H_1 & I_{N/2} \end{bmatrix} \cdot \begin{bmatrix} I_{N/2} & H_2 \\ & I_{N/2} \end{bmatrix} \cdot \begin{bmatrix} I_{N/2} & \\ H_3 & I_{N/2} \end{bmatrix} \tag{10}$$

where

$$H_1 = H_3 = \begin{bmatrix} & & & -\tan\frac{(N-1)\pi}{8N} \\ & & \iddots & \\ & -\tan\frac{3\pi}{8N} & & \\ -\tan\frac{\pi}{8N} & & & \end{bmatrix}.$$

$$H_2 = \begin{bmatrix} & & & \sin\frac{\pi}{4N} \\ & & \sin\frac{3\pi}{4N} & \\ & \iddots & & \\ \sin\frac{(N-1)\pi}{4N} & & & \end{bmatrix}$$

Thus, Equation (3) can be re-written as

$$C_N^{IV} = R_1 R_2 R_3 T_1 T_2 T_3 \tag{11}$$

Matrix S is defined as the product of R_3 and T_1, that is

$$S = R_3 T_1 = \begin{bmatrix} I_{N/2} & \\ H_3 + K_1 & I_{N/2} \end{bmatrix} \tag{12}$$

Obviously, S is also a lifting matrix. From (11) and (12), the final factorization formula for DCT-IV matrix is obtained as:

$$C_N^{IV} = R_1 R_2 S\ T_2 T_3 \tag{13}$$

Equation (13) indicates that integer DCT-IV transform can be realized by five lifting stages. Because these five lifting stages have identical structures, only the transform equations for one stage is listed here. It is very easy and straightforward to write down the transform equations for all the other stages. The first stage T_3 is used as an example.

Let $\boldsymbol{x} = [x(n)]_{n=0,1,\cdots,N-1}$ and $\boldsymbol{y} = [y(m)]_{m=0,1,\cdots,N-1}$ be the input and output $N \times 1$ integer vectors for the first transform stage:

$$\boldsymbol{y} = T_3 \boldsymbol{x} \tag{14}$$

vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ are divided from the center into two halves, that is

$$\boldsymbol{x}=\begin{bmatrix}\boldsymbol{x}_1\\ \boldsymbol{x}_2\end{bmatrix}\quad \boldsymbol{y}=\begin{bmatrix}\boldsymbol{y}_1\\ \boldsymbol{y}_2\end{bmatrix} \tag{15}$$

Using (9) and (15), (14) is re-written as

$$\begin{bmatrix}\boldsymbol{y}_1\\ \boldsymbol{y}_2\end{bmatrix}=\begin{bmatrix}I_{N/2} & \\ K_3 & I_{N/2}\end{bmatrix}\cdot\begin{bmatrix}\boldsymbol{x}_1\\ \boldsymbol{x}_2\end{bmatrix} \tag{16}$$

From (16), the time to frequency domain integer transform is:

$$\begin{aligned}\boldsymbol{y}_1 &= \boldsymbol{x}_1\\ \boldsymbol{y}_2 &= \lfloor K_3\cdot\boldsymbol{x}_1\rfloor+\boldsymbol{x}_2\end{aligned} \tag{17}$$

where $\lfloor * \rfloor$ denotes rounding operation. Figure 2 illustrates the method of transforming a digital signal from the time domain to the frequency domain using DCT-IV as the transformation function. x_1 *and* x_2 are two sub-blocks of a block of the input digital signal, z_1, z_2 and z_3 are intermediate signals, and y_1 and y_2 are corresponding sub-blocks of the output signal.

Similarly, the frequency to time domain transform is given by:

$$\begin{aligned}\boldsymbol{x}_1 &= \boldsymbol{y}_1\\ \boldsymbol{x}_2 &= \boldsymbol{y}_2-\lfloor K_3\cdot\boldsymbol{x}_1\rfloor\end{aligned} \tag{18}$$

where $\lfloor * \rfloor$ denotes rounding operation. Figure 3 illustrates the method of transforming a digital signal from the frequency domain to the time domain using DCT-IV as the transformation function.

Example of the method according to the invention based on an Integer-type DCT IV as the transformation function.

Another example of the method according to the invention is illustrated using the DWT-IV transformation function. The DWT-IV of a N-point real input sequence $x(n)$ is defined as follows:

$$y(m) = \sqrt{\frac{2}{N}} \sum_{n=0}^{N-1} x(n) \sin\left(\frac{\pi}{4} + \frac{(m+1/2)(n+1/2)2\pi}{N}\right) \quad m,n = 0,1,\cdots,N-1 \tag{19}$$

Let W_N^{IV} be the transformation matrix of DWT-IV, that is

$$W_N^{IV} = \sqrt{\frac{2}{N}} \left[\sin\left(\frac{\pi}{4} + \frac{(m+1/2)(n+1/2)2\pi}{N}\right)\right]_{m,n=0,1,\cdots,N-1} \tag{20}$$

The DWT-IV matrix can be factorized into the following form:

$$W_N^{IV} = R_N T_N P_N \tag{21}$$

R_N is a $N \times N$ rotation matrix

$$R_N = \frac{1}{\sqrt{2}} \begin{bmatrix} I_{N/2} & J_{N/2} \\ -J_{N/2} & I_{N/2} \end{bmatrix} \tag{22}$$

$I_{N/2}$ is the identity matrix of order N/2. $J_{N/2}$ is the counter identity matrix of order N/2 which has the following form:

$$J_{N/2} = \begin{bmatrix} & & & 1 \\ & & 1 & \\ & 1 & & \\ \iddots & & & \\ 1 & & & \end{bmatrix} \tag{23}$$

P_N is a $N \times N$ permutation matrix

$$P_N = \begin{bmatrix} I_{N/2} & \\ & J_{N/2} \end{bmatrix} \tag{24}$$

T is a $N \times N$ matrix given by

$$T = \frac{1}{\sqrt{2}} \begin{bmatrix} C_{N/2}^{IV} & -C_{N/2}^{IV} \\ C_{N/2}^{IV} D_{N/2} & C_{N/2}^{IV} D_{N/2} \end{bmatrix} \tag{25}$$

where $C_{N/2}^{IV}$ is the DCT-IV matrix of order N/2

$$C_{N/2}^{IV} = \sqrt{\frac{2}{(N/2)}} \left[\cos\left(\frac{(m+1/2)(n+1/2)\pi}{(N/2)} \right) \right]_{m,n=0,1,\cdots,N/2-1} \tag{26}$$

$D_{N/2}$ is an order N/2 diagonal matrix given by

$$D_{N/2} = \begin{bmatrix} 1 & & & & \\ & -1 & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & -1 \end{bmatrix} \tag{27}$$

Both R_N and T can be factorized into the product of three lifting matrices:

$$T = T_1 T_2 T_3 = \begin{bmatrix} I_{N/2} & \\ K_1 & I_{N/2} \end{bmatrix} \cdot \begin{bmatrix} D_{N/2} & K_2 \\ & I_{N/2} \end{bmatrix} \cdot \begin{bmatrix} I_{N/2} & \\ K_3 & I_{N/2} \end{bmatrix} \tag{28}$$

where $K_1 = \left(\sqrt{2} I_{N/2} - C_{N/2}^{IV} D_{N/2}\right) C_{N/2}^{IV}$ $K_2 = \dfrac{-C_{N/2}^{IV}}{\sqrt{2}}$ $K_3 = \sqrt{2} C_{N/2}^{IV} D_{N/2} - I_{N/2}$.

$$R_N = R_1 R_2 R_3 = \begin{bmatrix} I_{N/2} & \\ H_1 & I_{N/2} \end{bmatrix} \cdot \begin{bmatrix} I_{N/2} & H_2 \\ & I_{N/2} \end{bmatrix} \cdot \begin{bmatrix} I_{N/2} & \\ H_3 & I_{N/2} \end{bmatrix} \tag{29}$$

where

$$H_1 = H_3 = -\tan(\pi/8) \cdot J_{N/2} = \begin{bmatrix} & & & -0.414 \\ & & \cdot^{\cdot^{\cdot}} & \\ & -0.414 & & \\ -0.414 & & & \end{bmatrix}$$

$$H_2 = \sin(\pi/4) \cdot J_{N/2} = \begin{bmatrix} & & & 0.707 \\ & & 0.707 & \\ & \cdot^{\cdot^{\cdot}} & & \\ 0.707 & & & \end{bmatrix}$$

Thus, Equation (21) can be re-written as

$$C_N^{IV} = R_1 R_2 R_3 T_1 T_2 T_3 P_N \tag{30}$$

Matrix S as the product of R_3 and T_1, that is defined as:

$$S = R_3 T_1 = \begin{bmatrix} I_{N/2} & \\ H_3 + K_1 & I_{N/2} \end{bmatrix} \tag{31}$$

Obviously, S is also a lifting matrix. From (30) and (31), the final factorization formula for DWT-IV matrix is obtained as:

$$C_N^{IV} = R_1 R_2 S\ T_2 T_3 P_N \tag{32}$$

Equation (32) indicates that integer DWT-IV transform can be realized by five lifting stages plus a data shuffling stage defined by permutation matrix P_N. The data shuffling stage rearranges the components order in each input data block. According to P_N, the input data vector is rearranged in the following way: the first half of vector remains unchanged; the second half of vector is flipped – that is

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N/2} \\ x_{N/2+1} \\ \vdots \\ x_{N-1} \\ x_N \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N/2} \\ x_N \\ x_{N-1} \\ \vdots \\ x_{N/2+1} \end{bmatrix} \tag{33}$$

Because the five lifting stages have identical structures, only the transform equations for one stage is listed here. It is very easy and straightforward to write down the transform equations for all the other stages. The first stage T_3 is used as an example.

Let $\boldsymbol{x} = [x(n)]_{n=0,1,\cdots,N-1}$ and $\boldsymbol{y} = [y(m)]_{m=0,1,\cdots,N-1}$ be the input and output $N \times 1$ integer vectors for the first transform stage:

$$\boldsymbol{y} = T_3 \boldsymbol{x} \tag{34}$$

Vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ are divided from the center into two halves, that is

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{35}$$

Using (28) and (34), (33) is re-written as

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} I_{N/2} & \\ K_3 & I_{N/2} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{36}$$

From (36), the time to frequency domain integer transform is:

$$\begin{aligned} y_1 &= x_1 \\ y_2 &= \lfloor K_3 \cdot x_1 \rfloor + x_2 \end{aligned} \tag{37}$$

Figure 4 illustrates the method of transforming a digital signal from the time domain to the frequency domain using DCT-IV as the transformation function. x_1 *and* x_2 are two sub-blocks of a block of the input digital signal, z_1, z_2 and z_3 are intermediate signals, and y_1 and y_2 are corresponding sub-blocks of the output signal.

Similarly, the frequency to time domain integer transform is given by

$$\begin{aligned} x_1 &= y_1 \\ x_2 &= y_2 - \lfloor K_3 \cdot x_1 \rfloor \end{aligned} \tag{38}$$

Figure 5 illustrates the method of transforming a digital signal from the frequency domain to the time domain using DCT-IV as the transformation function

Conclusion of the two examples

From (17) and (18), there are *N/2* roundings in each lifting stage. Therefore, from (13), the total rounding number for the proposed integer DCT-IV algorithm is five times *N/2*, that is *2.5N*, which is significantly lower than $N.\log_2 N$ according to the state of the art. In (13), the majority of computation power is used in the four *N/2* point DCT-IV

subroutines $C^{IV}_{N/2}$, when N is a large value, e.g. $N = 1024$. Because the floating-point DCT-IV C^{IV}_N can be calculated using two half-length DCT-IV $C^{IV}_{N/2}$ plus pre- and post-rotations according to (3), the arithmetic complexity of the proposed integer DCT-IV is roughly estimated to be twice that of the floating-point DCT-IV.

A similar conclusion can be drawn for the integer DWT-IV transformation function, wherein five lifting stages are used.

In this invention, a method for realizing reversible transformation function, for example for integer type-IV DCT and DWT transformation function, is proposed. This method requires significantly reduced number of roundings for every block of N input samples. As a result, the approximation error is greatly reduced. The method according to the invention is low in computational complexity and modular in structure.

Reference

[1] R. Geiger, T. Sporer, J. Koller, K. Brandenburg, "Audio Coding based on Integer Transforms" *AES 111th Convention*, New York, USA, Sept. 2001

[2] H. S. Malvar, "Signal Processing with Lapped Transforms" *Artech House*, 1992

[3] "Coding of Moving Pictures and Audio: Workplan for Evaluation of Integer MDCT for FGS to Lossless Experimentation Framework" *ISO/IEC JTC 1/SC 29/WG 11 N5578*, Pattaya, Thailand, Mar. 2003

What is claimed is:

1. A method for transforming a digital signal from the time domain into the frequency domain using a transformation function comprising a transformation matrix, the digital signal is divided into a plurality of blocks, each block comprising a predefined number of data symbols, the method comprising

- dividing each block into two sub-blocks with equal number of data symbols,

- transforming the two sub-blocks by a transforming element, the transforming element comprises a plurality of lifting stages, wherein the two sub-blocks are received by a first lifting stage of the plurality of lifting stages as first and second input signals, and wherein each lifting stage comprises a transformation path, wherein the first input signal received by the lifting stage is processed by a domain transformer and a rounding unit, and is subsequently summed with the second input signal in a summation unit to result in an output signal; and wherein each subsequent lifting stage receives the output signal of the previous lifting stage as the first input signal, and the first input signal of the previous lifting stage as the second input signal, to generate the corresponding output signal of the subsequent stage, and wherein the number of lifting stages is defined by the number of lifting matrices determined by a process comprising

factorizing the transformation matrix into a string of first lifting matrices, each of which comprises two permutation sub-matrices in one diagonal, and a zero and a general sub-matrix in the other diagonal,

combining all such pairs of first lifting matrices being adjacent in the said string the respective permutation sub-matrices of which are identical and the general sub-matrices of which are at their corresponding positions, thereby generating second lifting matrices, wherein the number of stages corresponds to the number of the second lifting matrices.

2. The method of claim 1, wherein the transformation function is a DCT I transformation function, DCT II transformation function, DCT IV transformation function, FFT transformation function or DWT-IV transformation function.

3. The method of claim 1, wherein the step of factorizing the transformation matrix comprises decomposing the transformation matrix into a string of transformation sub-matrices and decomposing said transformation sub-matrices, if possible, into the said first lifting matrices.

4. The method of claim 2, wherein the step of factorizing the transformation matrix comprises decomposing the transformation matrix into a string of transformation sub-matrices and decomposing said transformation sub-matrices, if possible, into the said first lifting matrices.

5. The method of claim 3 or 4, wherein each of said decomposed matrices are further decomposed into three lifting matrices, resulting in nine first lifting matrices.

6. The method of claim 4, wherein the DCT IV matrix of the DCT IV transformation is decomposed into a pre-rotation matrix, a DCT IV transformation-based matrix and a post rotation matrix as transformation sub-matrices.

7. The method of claim 6, wherein the DCT IV transformation-based matrix and the post rotation matrix are combined into a T matrix, and the pre-rotation matrix and the T matrix are each decomposed into three first lifting matrices, resulting in six first lifting matrices.

8. The method of claim 7, wherein the last lifting matrix of the pre-rotation matrix is combined with the first lifting matrix of the T matrix, resulting in a total of five second lifting matrices.

9. A method according to any one of the preceding claims, wherein each element of the first and second lifting matrices is a matrix.

10. A device for transforming a digital signal from the time domain into the frequency domain using a transformation function comprising a transformation matrix, the digital signal is divided into a plurality of blocks, each block comprising a predefined number of data symbols, the device comprising

- a dividing unit for dividing each block into two sub-blocks with equal number of data symbols,

- a transformation unit for transforming the two sub-blocks by a transforming element, the transforming element comprises a plurality of lifting stages, wherein the two sub-blocks are received by a first lifting stage of the plurality of lifting stages as first and second input signals, and wherein each lifting stage comprises a transformation path, wherein the first input signal received by the lifting stage is processed by a domain transformer and a rounding unit, and is subsequently summed with the second input signal in a summation unit to result in an output signal; and wherein each subsequent lifting stage receives the output signal of the previous lifting stage as the first input signal, and the first input signal of the previous lifting stage as the second input signal, to generate the corresponding output signal of the subsequent stage, and wherein the number of lifting stages is defined by the number of lifting matrices determined by a process comprising

factorizing the transformation matrix into a sting of first lifting matrices, each of which comprises two permutation sub-matrices in one diagonal, and a zero and a general sub-matrix in the other diagonal,

combining all such pairs of first lifting matrices being adjacent in the said string the respective permutation sub-matrices of which are identical and the general sub-matrices of which are at their corresponding positions, thereby generating second lifting matrices, wherein the number of stages corresponds to the number of the second lifting matrices.

11. A computer readable medium, having a program recorded thereon, wherein the program is to make the computer execute a procedure for transforming a digital signal from the time domain into the frequency domain using a transformation function comprising a transformation matrix, the digital signal is divided into a plurality of blocks, each block comprising a predefined number of data symbols, the method comprising

- dividing each block into two sub-blocks with equal number of data symbols,

- transforming the two sub-blocks by a transforming element, the transforming element comprises a plurality of lifting stages, wherein the two sub-blocks are received by a first lifting stage of the plurality of lifting stages as first and second input signals, and wherein each lifting stage comprises a transformation path, wherein the first input signal received by the lifting stage is processed by a domain transformer and a rounding unit, and is subsequently summed with the second input signal in a summation unit to result in an output signal; and wherein each subsequent lifting stage receives the output signal of the previous lifting stage as the first input signal, and the first input signal of the previous lifting stage as the second input signal, to generate the corresponding output signal of the subsequent stage, and wherein the number of lifting stages is defined by the number of lifting matrices determined by a process comprising

factorizing the transformation matrix into a sting of first lifting matrices, each of which comprises two permutation sub-matrices in one diagonal, and a zero and a general sub-matrix in the other diagonal,

combining all such pairs of first lifting matrices being adjacent in the said string the respective permutation sub-matrices of which are identical and the general sub-matrices of which are at their corresponding positions, thereby generating second lifting matrices, wherein the number of stages corresponds to the number of the second lifting matrices.

12. A computer program element which is to make the computer execute a procedure for transforming a digital signal from the time domain into the frequency domain using a

transformation function comprising a transformation matrix, the digital signal is divided into a plurality of blocks, each block comprising a predefined number of data symbols, the method comprising

- dividing each block into two sub-blocks with equal number of data symbols,

- transforming the two sub-blocks by a transforming element, the transforming element comprises a plurality of lifting stages, wherein the two sub-blocks are received by a first lifting stage of the plurality of lifting stages as first and second input signals, and wherein each lifting stage comprises a transformation path, wherein the first input signal received by the lifting stage is processed by a domain transformer and a rounding unit, and is subsequently summed with the second input signal in a summation unit to result in an output signal; and wherein each subsequent lifting stage receives the output signal of the previous lifting stage as the first input signal, and the first input signal of the previous lifting stage as the second input signal, to generate the corresponding output signal of the subsequent stage, and wherein the number of lifting stages is defined by the number of lifting matrices determined by a process comprising

factorizing the transformation matrix into a sting of first lifting matrices, each of which comprises two permutation sub-matrices in one diagonal, and a zero and a general sub-matrix in the other diagonal,

combining all such pairs of first lifting matrices being adjacent in the said string the respective permutation sub-matrices of which are identical and the general sub-matrices of which are at their corresponding positions, thereby generating second lifting matrices, wherein the number of stages corresponds to the number of the second lifting matrices.

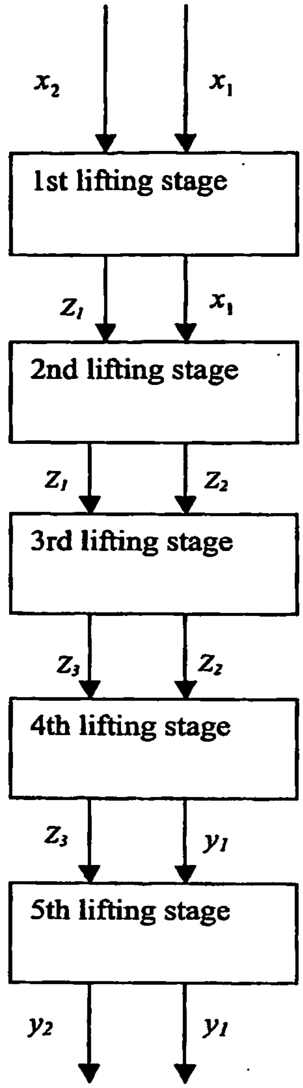
x_2
x_1
1st lifting stage
Z_1
x_1
2nd lifting stage
Z_1
Z_2
3rd lifting stage
Z_3
Z_2
4th lifting stage
Z_3
y_1
5th lifting stage
y_2
y_1

Figure 1.

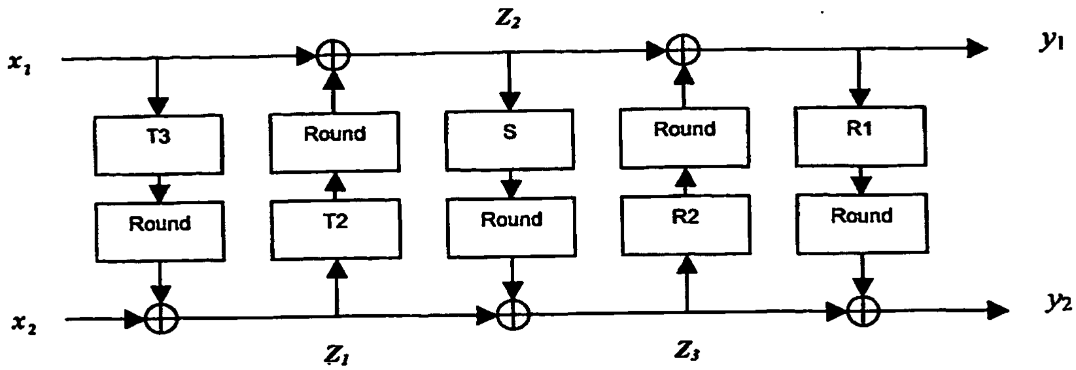

Figure 2. Forward IntDCT-IV flowchart

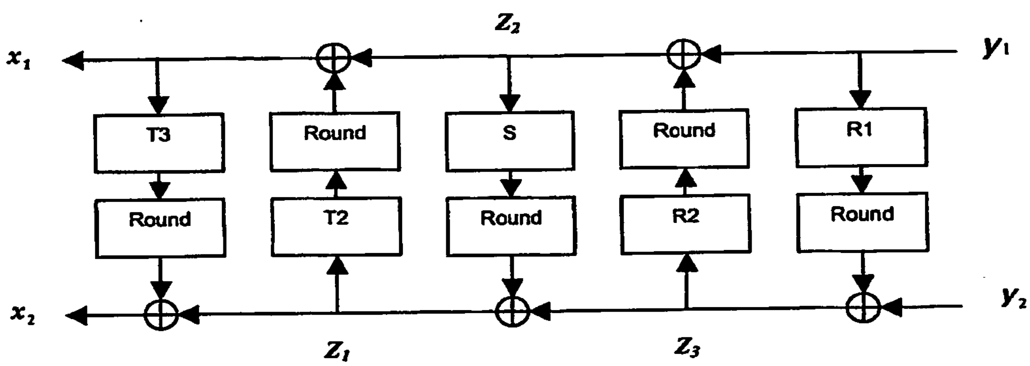

Figure 3. Inverse IntDCT-IV flowchart

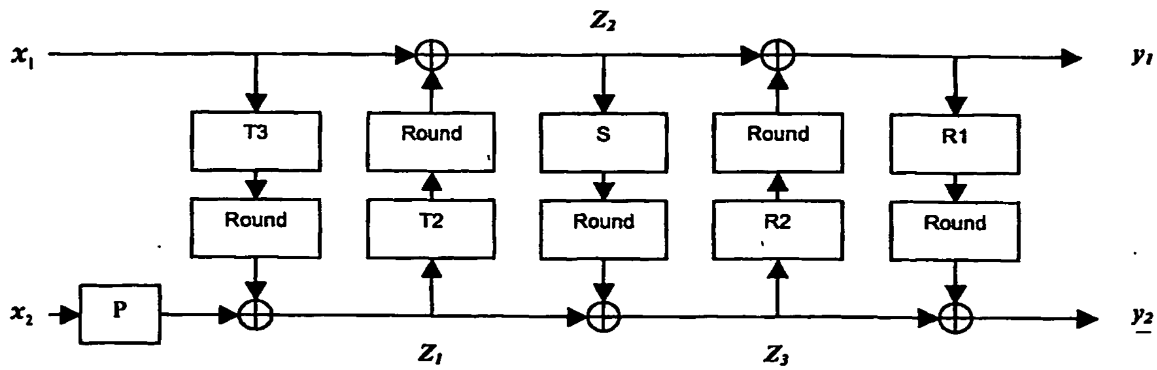

Figure 4. Forward IntDWT-IV flowchart

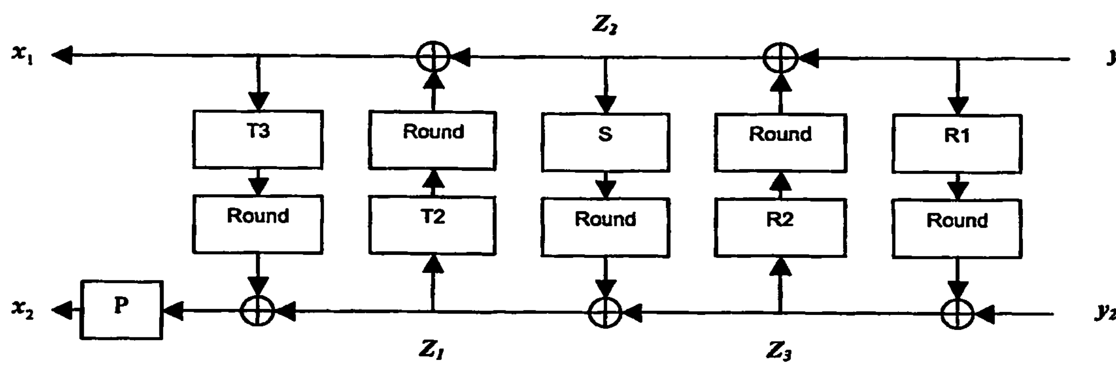

Figure 5. Inverse IntDWT-IV flowchart